Direct motion compensation in attenuation-corrected PET/CT and PET/MR reconstruction

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Motion in PET

Degraded resolution
Motion in PET

non motion corrected reconstruction
Degraded resolution
Motion in PET

Moving PET, static CT
Motion in PET

Moving PET, static CT

OK

attenuation artifacts
“Traditional” indirect approaches for Motion Compensated PET
“Traditional” indirect approaches for Motion Compensated PET

PET gate 1
CT gate 1
PET gate 2
CT gate 2
PET gate 3
CT gate 3
“Traditional” indirect approaches for Motion Compensated PET
“Traditional” indirect approaches for Motion Compensated PET

\[
\hat{f} = \arg \max_{f} p(g | \tilde{g}(f, \mu))
\]

PET gate 1
CT gate 1

PET gate 2
CT gate 2

PET gate 3
CT gate 3

Data

MLEM Recon.
“Traditional” indirect approaches for Motion Compensated PET

\[ f = \arg \max_f p(g \mid \tilde{g}(f, \mu)) \]

\[ \tilde{f} = \arg \max_{\tilde{f}} \frac{p(g \mid \tilde{g}(f, \mu))}{\tilde{g}(f, \mu)} \]

\[ \hat{f} = \arg \max_{\hat{f}} p(g \mid \tilde{g}(f, \mu)) \]

Data

PET gate 1

CT gate 1

PET gate 2

CT gate 2

PET gate 3

CT gate 3

MLEM Recon.

Registration

Warped images
“Traditional” indirect approaches for Motion Compensated PET

PET gate 1
CT gate 1

PET gate 2
CT gate 2

PET gate 3
CT gate 3

Data
Registration
Averaging
(consolidation)

MLEM
Recon.
Warped
images

\[ \hat{f} = \arg \max_f p(g | \tilde{g}(f, \mu)) \]

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M. Dawood, MICCAI 2008
I. Polycarpou, Med. Phys, 2012
“Traditional” indirect approaches for Motion Compensated PET

\[ f = \arg \max_{f} p(g \mid \tilde{g}(f, \mu)) \]

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“Traditional” indirect approaches for Motion Compensated PET

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\[ \hat{f} = \arg \max_f p(g | \tilde{g}(f, \mu)) \]

\[ \hat{f} = \arg \max_f p(g | \tilde{g}(f, \mu)) \]

Data

Registration and extraction of motion field \( W \)

PET gate 1
CT gate 1
PET gate 2
CT gate 2
PET gate 3
CT gate 3

MLEM Recon.
“Traditional” indirect approaches for Motion Compensated PET

\[ f = \arg \max_f p(g \mid \tilde{g}(f, \mu)) \]

\[ \hat{f} = \arg \max_f p(g \mid \tilde{g}(f, \mu)) \]

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R. Manjeshwar, IEEE ISBI 2006 (motion from CT)
“Traditional” indirect approaches for Motion Compensated PET

- Requires attenuation map for each gate
- Registration with noisy images (each gate is reconstructed independently) is difficult
- Post-reconstruction averaging introduces bias due to the nonlinearity of the maximum-likelihood
Aim of this work

Motion compensated PET reconstruction

To jointly estimate the activity distribution and the motion from the raw PET data, using a single CT/MRI attenuation-map, to provide motion-free and attenuation artifact-free images.
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Motion compensated PET reconstruction
To jointly estimate the activity distribution and the motion from the raw PET data, using a single CT/MRI attenuation-map, to provide motion-free and attenuation artifact-free images.

Our approach

- Direct maximisation of the likelihood of the (gated) PET data
- No post-reconstruction image registration
Plan
Forward model with motion

Generic activity $f$

Generic attenuation $\mu$
Forward model with motion

Generic activity $f$

(same motion for $f$ and $\mu$)

Generic attenuation $\mu$

$W_1$
Forward model with motion

Generic activity \( f \)

(same motion for \( f \) and \( \mu \))

Generic attenuation \( \mu \)

\[ W_1 \]

\[ W_2 \]
Forward model with motion

Generic activity $f$

(same motion for $f$ and $\mu$)

Generic attenuation $\mu$
Forward model with motion

\[ g_1 = H(W_1 \mu) W_1 f \]

\[ g_2 = H(W_2 \mu) W_2 f \]

\[ g_3 = H(W_3 \mu) W_3 f \]

Forward model

- $f$: reference activity distribution
- $\mu$: attenuation map ($\mu$-map)
- $\bar{g}$: expected counts
- $\mathcal{W}_t f$: deformed activity
- $\mathcal{W}_t \mu$: deformed attenuation
- $P(\mathcal{W}_t \mu) \triangleq e^{-R_{\mathcal{W}_t} \mu} P$: attn-corrected PET system matrix with deformed attenuation

Reconstruction

- $g_t$: observed counts at gate $t$
- $\hat{f}$: estimated activity; $\hat{\mathcal{W}}_t$: estimated motion; $L$: Poisson log-likelihood

$$\left( \hat{f}, \hat{\mathcal{W}} \right) = \arg \max_{f \geq 0, \mathcal{W}} \sum_{t=1}^{\text{#gates}} L(g_t \mid \bar{g}_t(f, \mu, \mathcal{W}_t))$$
Joint Motion Estimation/Image Reconstruction (JRM) by Maximum Likelihood

Update in $f$
(Penalised) MLEM with motion and attenuation corrected system matrix $[P(W\mu)W]$

Update in $W$
Quasi-Newton linesearch:

$$W_t^{\text{new}} = W_t^{\text{old}} - \alpha^* B \nabla W_t L$$

with

$$\alpha^* = \arg \max_{\alpha > 0} L(g_t \mid \tilde{g}_t(f, \mu, W_t^{\text{old}} - \alpha^* B \nabla W_t L))$$

and $B$ is a L-BFGS approximation of the inverse Hessian of $L$ in $W$
Joint Motion Estimation/Image Reconstruction (JRM) by Maximum Likelihood

PET gate 1

PET gate 2

PET gate 3
Joint Motion Estimation/Image Reconstruction (JRM) by Maximum Likelihood

PET gate 1
Single CT mu-map
PET gate 2
PET gate 3
Data
Joint Motion Estimation/Image Reconstruction (JRM) by Maximum Likelihood

\[
(f, \hat{W}) = \arg \max_{f, \hat{W}} p(g | \tilde{g}(Wf, W\mu))
\]
Joint Motion Estimation/Image Reconstruction (JRM) by Maximum Likelihood

\[
(f, \tilde{W}) = \arg \max_{f, W} p(g | \tilde{g}(Wf, W\mu))
\]

Deformation field \( W \)
Joint Motion Estimation/Image Reconstruction (JRM) by Maximum Likelihood

\[
(f, \hat{W}) = \arg \max_{f, W} p(g | \tilde{g}(W f, W \mu))
\]

Data

PET gate 1

PET gate 2

PET gate 3

Single CT mu-map

Deformation field \( W \)

Motion comp. PET image
Joint Motion Estimation/Image Reconstruction (JRM) by Maximum Likelihood

\[ (f, \hat{W}) = \arg \max_{f, W} p(g | \tilde{g}(Wf, W\mu)) \]

- PET gate 1
- PET gate 2
- PET gate 3
- Data
- Single CT mu-map
- Recon. PET gate 1
- Recon. PET gate 2
- Recon. PET gate 3

Deformation field \( W \)

Motion comp. PET image

Deformation with \( W \)
Joint Motion Estimation/Image Reconstruction (JRM) by Maximum Likelihood

\[ (f, \hat{W}) = \arg \max_{f, W} p(g | \bar{g}(WF, W\mu)) \]
Advantages of JRM

- Only one $\mu$-map needed
- in PET/CT: less X-ray dose for the patient
- Gated $\mu$-map delivered for free
- One single activity image $f$ estimated from the entire PET sequence: maximisation of signal to noise ratio
- Reconstructed PET images are spatially matched to corresponding $\mu$-map: no attenuation artifacts
- JRM achieves lowest variance for a given bias level (maximum likelihood)
How to choose $\mu$?

- $\mu$: given attenuation map
How to choose $\mu$?

- $\mu$: given attenuation map
- $\tilde{\mu} = M\mu$: deformed version of $\mu$ (mismatch)
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- $\tilde{\mu} = M\mu$: deformed version of $\mu$ (mismatch)

$$
\bar{g}(f, W, \mu) = P(W\mu)Wf + r = P(WM^{-1}M\mu)W\bar{M}^{-1}Mf + r
$$

$$
= \bar{g}(\bar{M}f, WM^{-1}, \tilde{\mu})
$$
How to choose $\mu$?

- $\mu$: given attenuation map
- $\tilde{\mu} = M\mu$: deformed version of $\mu$ (mismatch)

$$
\bar{g}(f, \mathcal{W}, \mu) = P(\mathcal{W}\mu)\mathcal{W}f + r = P(\mathcal{W}M^{-1}\mathcal{M}\mu)\mathcal{W}M^{-1}Mf + r
$$
$$
= \bar{g}(Mf, \mathcal{W}M^{-1}, \tilde{\mu})
$$

- If

$$(\tilde{f}, \tilde{\mathcal{W}}) = \arg\max_{f, \mathcal{W}} L(g \mid \bar{g}(f, \mathcal{W}, \tilde{\mu}))$$

then

$$(\hat{f}, \hat{\mathcal{W}}) = (M^{-1}\tilde{f}, \hat{\mathcal{W}}M) = \arg\max_{f, \mathcal{W}} L(g \mid \bar{g}(f, \mathcal{W}, \mu))$$

$\hat{\mathcal{W}}\hat{f} = \hat{\mathcal{W}}\tilde{f}$ and $\hat{\mathcal{W}}\tilde{\mu} = \hat{\mathcal{W}}\mu$
How to choose $\mu$?

- $\mu$: given attenuation map
- $\tilde{\mu} = M\mu$: deformed version of $\mu$ (mismatch)

$$
\bar{g}(f, W, \mu) = P(W\mu)WF + r = P(WM^{-1}M\mu)WM^{-1}Mf + r
$$

$$
\bar{g}(Mf, WM^{-1}, \tilde{\mu})
$$

- If

$$(\tilde{f}, \tilde{W}) = \arg\max_{f, W} L(g | \bar{g}(f, W, \tilde{\mu}))$$

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$$(\hat{f}, \hat{W}) = (M^{-1}\tilde{f}, \tilde{W}M) = \arg\max_{f, W} L(g | \bar{g}(f, W, \mu))$$

$$
\hat{W}\tilde{f} = \tilde{W}\hat{f} \quad \text{and} \quad \hat{W}\tilde{\mu} = \tilde{W}\mu
$$

- JRM reconstructs $f$ in the $\tilde{\mu}$-space, and warps $f$ and $\tilde{\mu}$ by combining the mis-alignment deformation with the motion
How to choose $\mu$?

- $\mu$: given attenuation map
- $\tilde{\mu} = \mathcal{M}\mu$: deformed version of $\mu$ (mismatch)

$$
\bar{g}(f, \mathcal{W}, \mu) = P(\mathcal{W}\mu)\mathcal{W}f + r = P(\underbrace{\mathcal{W}\mathcal{M}^{-1}\mathcal{M}\mu}_{\text{mismatch}})\underbrace{\mathcal{W}\mathcal{M}^{-1}\mathcal{M}f}_{\text{mis-alignment}} + r

= \bar{g}(\mathcal{M}f, \mathcal{W}\mathcal{M}^{-1}, \tilde{\mu})
$$

- If

$$
(f, \tilde{\mathcal{W}}) = \arg \max_{f, \mathcal{W}} L(g \mid \bar{g}(f, \mathcal{W}, \tilde{\mu}))
$$

then

$$(\hat{f}, \hat{\mathcal{W}}) = (\mathcal{M}^{-1}\tilde{f}, \tilde{\mathcal{W}}\mathcal{M}) = \arg \max_{f, \mathcal{W}} L(g \mid \bar{g}(f, \mathcal{W}, \mu))$$

$$
\begin{align*}
\hat{\mathcal{W}}\hat{f} &= \hat{\mathcal{W}}\tilde{f} & \text{and} & & \hat{\mathcal{W}}\tilde{\mu} &= \hat{\mathcal{W}}\mu
\end{align*}
$$

- JRM reconstructs $f$ in the $\tilde{\mu}$-space, and warps $f$ and $\tilde{\mu}$ by combining the mis-alignment deformation with the motion
- $\mu$ does not need to be aligned with the PET data
Plan
Patient Data: GE Discovery STE (RPM amplitude gating)

All gates, no MC

JRM
Patient Data: GE Discovery STE (RPM amplitude gating)

- gated recon.
- gated CT (cine CT)

JRM

JRM warped $\mu$

XCAT simulations

Activity phantom

\( \mu \) (aligned with gate 1) \( \tilde{\mu} \) (mis-aligned with all gates)

\( \mu - \tilde{\mu} \)
XCAT simulations

Reco. gates with $\mu$: $\hat{\mathcal{W}}_t \hat{f}$

Reco. gates with $\tilde{\mu}$: $\tilde{\mathcal{W}}_t \tilde{f}$

True activity
XCAT simulations

warped $\mu$: $\widehat{\mathcal{W}}_t \mu$

warped $\tilde{\mu}$: $\widetilde{\mathcal{W}}_t \tilde{\mu}$

True attenuation
**XCAT simulations**

Mean square error vs variance

JRM1: aligned \(\mu\)-map; JRM2: misaligned \(\mu\)-map; PRRC: post-reconstruction registration and consolidation
\( \mu \)-map realignment: standard PET

\[ \tilde{\mathcal{W}}\tilde{f} \]

\[ \tilde{\mathcal{W}}\tilde{\mu} - \mu^* \]

\( \mu \)-map realignment: standard PET

\( \tilde{\mathcal{W}} f \)

\( \tilde{\mathcal{W}} \tilde{\mu} - \mu^* \)

\( \mu \)-map realignment: time of flight PET

\[ \mathcal{W}\tilde{f} \]

\[ \mathcal{W}\tilde{\mu} - \mu^* \]

\( \mu \)-map realignment: time of flight PET

\[ \tilde{\mathcal{W}} \tilde{f} \]

\[ \tilde{\mathcal{W}} \tilde{\mu} - \mu^* \]

Plan
Challenges of PET/MR and direct motion compensation

The MRI-derived $\mu$-map...

- is obtained by segmentation into classes (soft tissues, air, etc.)
- may have “wrong” $\mu$-values: $\mu^\text{MR} \neq \mathcal{M}\mu^\text{true}$
- contains truncated features because of the limited FoV, e.g. the arms: non-diffeomorphic mismatch

Interrogation

Since JRM “adapts” the motion field so that the $\mu$-map matches the data, how will the algorithm behave with a deteriorated $\mu$-map?
Patient Data: Siemens mMR (PCA gating)

MRI derived $\mu$-map

Gate 1 recon. (no AC)

No motion corrected recon

JRM recon.
Patient Data: Siemens mMR (PCA gating)

No motion corrected recon.  

JRM recon.
XCAT simulations

true act.

true attn

“MRI” $\mu$-map

“MRI” $\mu$-map with trunc. arms
XCAT simulations

Reconstruction using

- Correct attenuation in the lungs
- Zero attenuation in the lungs
- Double attenuation in the lungs
- Truncated arms
XCAT simulations: correct lungs attenuation value

Recon. act.  Warped attn.

true act.  true attn
XCAT simulations: zero attenuation in the lungs

Recon. act.  Warped attn.

true act.  true attn
XCAT simulations: double attenuation in the lungs

Recon. act.  Warped attn.

true act.  true attn
XCAT simulations: double attenuation in the lungs

Recon. act.  Warped attn.

true act.  true attn
XCAT simulations: double attenuation in the lungs

“MRI” $\mu$-map with trunc. arms

After deformation to gate 1

Truth

JRM estimated a motion that stretches the arms to fit the data.
Plan
Conclusion

- JRM by maximum likelihood can jointly reconstruct activity and motion from gated PET and a single $\mu$-map: reduction of the patient X-ray dose in PET/CT.
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Conclusion

- JRM by maximum likelihood can jointly reconstruct activity and motion from gated PET and a single $\mu$-map: reduction of the patient X-ray dose in PET/CT.
- The $\mu$-map does not need to be aligned with the PET: a breath-held $\mu$-map can be used.
- The estimated motion applied to the $\mu$-map returns images similar to gated CT $\mu$-map (we obtain gated $\mu$-map for free).
Conclusion

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- JRM achieves better quantitative results compared to indirect methods (PRRC) that use gated CT.
Conclusion

- JRM by maximum likelihood can jointly reconstruct activity and motion from gated PET and a single $\mu$-map: reduction of the patient X-ray dose in PET/CT.
- The $\mu$-map does not need to be aligned with the PET: a breath-held $\mu$-map can be used.
- The estimated motion applied to the $\mu$-map returns images similar to gated CT $\mu$-map (we obtain gated $\mu$-map for free).
- JRM achieves better quantitative results compared to indirect methods (PRRC) that use gated CT.
- JRM seems to work with an MRI-derived $\mu$-map, and may correct for missing features (e.g. arms).
Further work

- Finding an alternative to line-search (costly)
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- Investigate priors
  - Image priors
  - Specific motion models for respiration
- TOF-PET (work in progress)
- Gated JRM Kinetic imaging with non-rigid motion (respiratory and cardiac)
- Gated SPECT
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