Joint estimation of activity and attenuation in TOF-PET

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• Attenuation Correction in PET
• Time of Flight (TOF) PET
  • Data collection
• Statistical Methods for Attenuation Correction in TOF-PET
  – MLAA
  – MLACF
  – MLRR
• Extension to Listmode
• Conclusion
• Transmission scans
  – Problem: cumbersome, and very time consuming

• High quality CT scans,
  – Facilitated by hybrid PET/CT scanners
  – Attenuation correction: projections of the “adjusted CT” (511 keV photon energy).
  – Problem: between-scan and in-scan motion.
    • PET and CT data are acquired sequentially.
    • Long acquisition time for the PET compared to the CT.

• Emission data (?!)
  – Useful: anatomical information not available, acquisition not possible
  – Joint activity and attenuation estimation

V. Panin, et al. 2011 NSSMIC.
• **Consistency Conditions**
  - Analytical Consistency Conditions
    • Natterer, *Inverse Problems* 1993
    • Welch et al, *IEEE TMI* 1997
  - Discrete Consistency Conditions

• **Iterative Schemes**
  - Can Time of Flight (TOF) improve the joint activity and attenuation reconstructions?!
    • De Pierro et al, *IEEE TNS* 2007
    • Salomon et al, *SNM* 2009
  - POCS, Least Squares
    • Censor et al, *IEEE TNS* 1979
    • Panin et al, *IEEE TNS* 2001

Cross-talk!
- Structures not preserved
- NOT quantitative
• Difference in photon arrival ($t_2 - t_1$)

• Time uncertainty ($\Delta t$)
  – Mainly due to detector properties
• Spatial uncertainty ($\Delta l = c \times \Delta t / 2$)

• Modeled by a Gaussian distribution
  – FWHM equal to spatial uncertainty

• Commercial TOF-PET systems
  • Biograph mCT: $\Delta t \approx 550$ ps $\leftrightarrow \Delta l \approx 8.25$ cm
  • Signa: $\Delta t \approx 400$ ps $\leftrightarrow \Delta l \approx 6.00$ cm
Tracer distribution
Activity
\(\lambda\)

Positron Emission Tomography (PET)
– *Data Collection with TOF*
Positron Emission Tomography (PET) – Data Collection with TOF

TOF Emission Data

\[ Y_t(\lambda, \mu) = P_t(\lambda)e^{-P(\mu)} \]

Reconstruction problem: Find \( \lambda \)
Consistency Conditions

Can Time of Flight (TOF) improve the joint activity and attenuation reconstructions?!

- Yes, very significantly! 😊

Iterative Schemes

- Maximum Likelihood
  - Nuyts et al, IEEE TMI 1999
  - Laymon et al, IEEE NSS-MIC 2004
  - De Pierro et al, IEEE TNS 2007
  - Salomon et al, SNM 2009
- POCS, Least Squares
  - Censor et al, IEEE TNS 1979
  - Panin et al, IEEE TNS 2001

Cross-talk!
- Structures not preserved
- NOT quantitative
Can Time of Flight (TOF) improve the joint activity and attenuation reconstructions?!

- Yes, very significantly! 😊
- But the reconstructions are NOT quantitative! 😞
Joint Activity and Attenuation Estimation
– Statistical methods, MLAA

\[ Y_t(\lambda, \mu) = P_t(\lambda)e^{-P(\mu)} \]

\[ [\hat{\lambda}_{mle}, \hat{\mu}_{mle}] = \arg\max_{\lambda, \mu} L([\lambda, \mu], Y) \]

MLEM (Maximum Likelihood Expectation Maximization)

1: \( \mu^n, \lambda^{n+1} = \arg\text{inc}_{\lambda} L([\lambda, \mu^n], Y) \)

2: \( \lambda^{n+1}, \mu^{n+1} = \arg\text{inc}_{\mu} L([\lambda^{n+1}, \mu], Y) \)

MLTR (Maximum Likelihood algorithm for Transmission Tomography)

Joint Activity and Attenuation Estimation
– Statistical methods, MLAA

Scale?
– Enforcing expected tissue attenuation during reconstruction!
  • Tissue: 0.096 cm$^{-1}$
  • Bone: 0.170 cm$^{-1}$

* A. Rezaei, M. Defrise, G. Bal, C. Michel, M. Conti, C. Watson, and J. Nuyts,
“Simultaneous Reconstruction of Activity and Attenuation in Time-of-Flight PET”,
• Thorax scan
  – Duration: 5 minute scan
  – Tracer: 570 MBq $^{18}$F-FDG
  – Siemens Biograph mCT PET/CT

• MLAA
  – Scaled to have expected tissue attenuation

* Data courtesy of M. Conti from Siemens Medical Solutions, Molecular Imaging, Knoxville, TN 37932 US
Joint Activity and Attenuation Estimation

– Statistical methods, MLACF

\[ Y_t(\lambda, A) = P_t(\lambda)A \]

\[ [\hat{\lambda}_{mle}, \hat{A}_{mle}] = \arg\max_{\lambda, A} L([\lambda, A], Y) \]

MLEM (Maximum Likelihood Expectation Maximization)

1: \[ A^n, \lambda^{n+1} = \arg\text{inc} L([\hat{\lambda}, A^n], Y) \]

2: \[ \lambda^{n+1}, A^{n+1} = \arg\text{inc} L([\lambda^{n+1}, A], Y) \]

ACF update


Joint Activity and Attenuation Estimation
- **Statistical methods, MLACF**

Scale?
- More complex!
- Post-reconstruction of ACFs
- Addition of a vial of known activity?


MLACF

- Patient data

- Thorax scan
  - Duration: 4 minute scan
  - Tracer: 296 MBq $^{18}$F-FDG
  - Siemens Biograph mCT PET/CT

- MLAA
  - Scaled by total amount of activity

- MLACF
  - Scaled by total amount of activity

* Data courtesy of S.Stroobants, S.Staelens and M.Lambrechts from Universiteit Antwerpen.
MLACF
– Brain scan data

• Brain scan
  – Duration: 5 minute scan
  – Tracer: 296 MBq $^{18}$F-FDG
  – Siemens Biograph mCT PET/CT

• MLAA
  – Scaled by expected tissue attenuation

• MLACF
  – Scaled by total amount of activity
  – Attenuation estimate regularized

* Data courtesy of S.Stroobants, S.Staelens and M.Lambrechts from Universiteit Antwerpen.
MLACF

- Brain scan data
Joint Activity and Attenuation Estimation
– Statistical methods, MLRR

\[
Y_t(\lambda, M) = P_t(\lambda) e^{-P(M^o\mu)}
\]

\[
[\hat{\lambda}_{mle}, \hat{M}_{mle}] = \arg\max_{\lambda, M} L([\lambda, M], Y)
\]

MLEM (Maximum Likelihood Expectation Maximization)

1: \(M^n, \lambda^{n+1} = \arg\max_{\lambda} L([\lambda, M^n], Y)\)

2: \(\lambda^{n+1}, M^{n+1} = \arg\max_{M} L([\lambda^{n+1}, M], Y)\)

MLTR + demons

* A. Rezaei, C. Michel, M. E. Casey, and J. Nuyts.
“Simultaneous reconstruction of the activity image and registration of the CT image in TOF-PET.”
Joint Activity and Attenuation Estimation
– Statistical methods, MLRR

Scale?
– Automatically fixed!

• Thorax scan
  – Duration: 4 minute scan
  – Tracer: 296 MBq $^{18}$F-FDG
  – Siemens Biograph mCT PET/CT

• MLAA
  – scaled to have expected tissue attenuation

• MLACF
  – scaled by total amount of activity

• MLRR
  – No scaling required

* Data courtesy of S.Stroobants, S.Staelens and M.Lambrechts from Universiteit Antwerpen.
Extension to Listmode - MLAA

- Hoffman brain scan

MLACF 10:12

UMAP

TOF-MLEM

MLAA – ATT.

MLAA – ACT.
Extension to Listmode - MLACF
– Hoffman brain scan

\[
\lambda_j^{(n+1)} = \frac{\lambda_j^{(n)}}{\sum_i c_{ij} \frac{y_i}{p_i^{(n)}}} \sum_{it} c_{ijt} \frac{y_{it}}{p_{it}^{(n)}}
\]
Extension to Listmode - MLACF
– Hoffman brain scan & Motion

- 10 min scan with continuous motion
- M Bickell et al. “Rigid motion correction of PET and CT for a PET/CT scanner”, MIC2015, M5DP-124
• 10 min scan with continuous motion
• M Bickell et al. “Rigid motion correction of PET and CT for a PET/CT scanner”, MIC2015, **M5DP-124**
• **MLACF**: no time-averaged sensitivity image needed
• With Time-of-Flight PET data,
  – Simultaneous activity and attenuation estimation is possible!
  – Constant/Scale correction technique needed

• MLAA
  – Scale correction: expected tissue attenuation!

• MLACF
  – Fast compared to MLAA/MLRR
  – Scale correction: ?!

• MLRR
  – Makes use of high CT quality
  – Scale correction: automatically solved
• Joint Estimation methods in Gated TOF-PET
  – Mitigate the longstanding problem of attenuation correction
  – Extended MLACF in a Fully4D reconstructions framework

• Future work
  – Tracers with more focal activity uptake
  – Analysis of different flavors of JE methods
    • E.g. MLACF with Poisson or Gaussian ACF model, sMLACF, …
  – More quantitative studies on JE methods
    • MLAA, MLACF and MLRR
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A Quantitative study
– Monte Carlo Simulation

• Activity recons. scaled to have “known” total activity

• Monte Carlo simulation
  – NEMA-like phantom
  – Siemens Biograph mCT specs.
  – No scatter in emission data
A Quantitative study
  – Monte Carlo Simulation

- Activity recons. scaled to have “known” total activity
- Monte Carlo simulation
  - NEMA-like phantom
  - Siemens Biograph mCT specs.
  - With scatter in emission data

![Graph showing scale comparison between different methods]
- **MLTR** (Maximum Likelihood Transmission Reconstruction)

**Sinogram**

- Attenuation image
- Voxel-LOR intersection length
- Measured data

\[
\mu_j^{(n+1)} = \mu_j^{(n)} - \frac{\sum_i l_{ij} (y_i - \bar{y}_i^{(n+1)})}{\sum_i l_{ij} \bar{y}_i^{(n+1)} \sum_\xi l_{i\xi}}
\]

**Listmode**

- Measured LOR index

\[
\mu_j^{(n+1)} = \mu_j^{(n)} - \frac{\sum_i l_{im,j} - \sum_i l_{ij} \bar{y}_i^{(n+1)}}{\sum_i l_{ij} \bar{y}_i^{(n+1)} \sum_\xi l_{i\xi}}
\]

- Expected counts

\[
\bar{y}_i^{(n)} = a_i^{(n)} p_i^{(n)}
\]

- LOR index
- Attenuation
- Activity projection

**ISRA-like algorithm**

- Image Space Reconstruction Algorithm
- Back-projection of all events
- Subsets on all possible LORS
Listmode MLAA: MLTR + MLEM

- **MLAA** (Maximum Likelihood Activity and Attenuation reconstruction)

### Sinogram

- Attenuation image
- Voxel-LOR intersection length
- Measured data

\[
\mu_j^{(n+1)} = \mu_j^{(n)} - \frac{\sum_i l_{ij} (y_i - y_i^{(n+1)}))}{\sum_i l_{ij} y_i^{(n+1)} \sum_\xi l_{ij}}
\]

- Voxel index
- Activity image
- Voxel-LOR sensitivity

\[
\lambda_j^{(n+1)} = \frac{\lambda_j^{(n)}}{\sum_i c_{ij} a_i^{(n)}} \sum_{it} c_{ijt} \frac{y_{it}}{p_{it}}
\]

### Listmode

- Measured LOR index

\[
\mu_j^{(n-1)} = \mu_j^{(n)} - \frac{\sum_{i=1}^m l_{imj} - \sum_i l_{ij} y_i^{(n-1)}}{\sum_i l_{ij} y_i^{(n-1)} \sum_\xi l_{ij}}
\]

- Expected counts

\[
\bar{y}_i^{(n)} = a_i^{(n)} p_i^{(n)}
\]

- Activity projection

- LOR index
- Attenuation

- 2 sets of subsets in MLEM
- 3:1 attenuation-to-activity updates
• **MLACF** *(Maximum Likelihood Attenuation Correction Factors)*

\[
\lambda_j^{(n+1)} = \frac{\lambda_j^{(n)}}{\sum_i c_{ij} \frac{y_i}{p_i^{(n)}}} \sum_{it} c_{ijt} \frac{y_{it}}{p_{it}}
\]

\[
\lambda_j^{(n+1)} = \frac{\lambda_j^{(n)}}{\sum_{i_m} c_{i_m j} \frac{1}{p_{i_m}^{(n)}}} \sum_{i_m t_m} c_{i_m j t_m} \frac{1}{p_{i_m t_m}}
\]

- Back-projections are matched
- **ONLY** Sensitivity of measured events needed
- ACFs are not computed explicitly

**Listmode**

\[
y_i^{(n)} = a_i^{(n)} p_i^{(n)}
\]